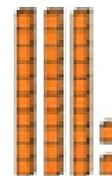


$$\begin{array}{ccc} \text{+10} & \text{+2} & \\ \hline 23 & 33 & 35 \end{array}$$



Tens	Ones
3	2

342 × 7 becomes

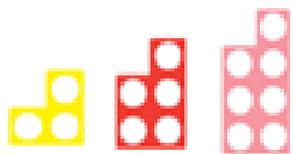
$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \hline 21 \end{array}$$

Mathematics at The Federation of Fairfield and Colneis

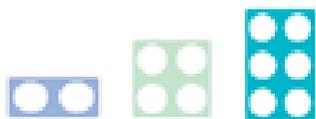


£10.00	
£6.74	?

$$\begin{array}{r} 374 \\ + 18 \\ \hline 392 \\ \hline 1 \end{array}$$



Odd



Even

$$\begin{array}{r} 36 \text{ r}4 \\ 7 \overline{) 256} \\ \underline{21} \\ 46 \\ \underline{42} \\ 4 \end{array}$$

$$\begin{array}{r} 36 \text{ r}4 \\ 7 \overline{) 256} \end{array}$$



Mathematics at Fairfield and Colneis

This document has been produced to share with staff, governors, parents and any visitors to our schools who have an interest in how our pupils learn mathematics.

We firmly believe that with the correct provision **ALL** pupils can make progress, achieve and enjoy mathematics. The whole school 'Growth Mindset' approach, encourages resilience, independence and self-belief.

The National Curriculum 2013 states that:

'The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage.

Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content.

Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.'

We have high expectations and will provide appropriate challenge and support for all.

Using the National curriculum objectives for each year group, the planning process identifies the small steps needed for **all** pupils to access the learning and achieve the objectives. Planning also considers the misunderstandings that may arise in some of the more complex concepts and the support that may be needed for pupils that do not grasp it quickly. Similarly, for those who grasp the concept rapidly, planning includes challenging questions and learning tasks that require a deeper exploration. Pupils are expected to be able to prove their answers in more than one way, explore different possibilities, verbalise thinking methods and reason concisely, using precise mathematical language.

Whole class learning

Within a maths lesson you might see:

- Whole class discussion and partner talk to aid reasoning and conjecturing, exploring how answers were obtained, why the method/strategy worked and what was the most effective.
- Conceptual variation – varied concrete and pictorial representations to build up strong mental images and deepen understanding of more abstract concepts.
- Written calculations are used alongside images and representations, and then questioning and discussions are used to make the connection between the written calculation and the resources.
- Practical problem solving / time for intelligent practice and consolidation and opportunities to practice and apply the skill being taught in increasingly complex ways.
- Procedural variation – looking for similarities and differences, patterns and connections. This is used to highlight the key learning point.
- Questioning so that teachers can assess the level of understanding and adapt as necessary
- Precise mathematical language, using full sentences is used by the teachers and modelled so that children become confident when conjecturing and reasoning.
- A wide range of resources are used consistently through all Key Stages to support understanding, the ability to reason and to develop a sense of enjoyment and curiosity about the subject.

Planning and provision

- Lessons are engaging and purposeful, providing opportunity for exploration, documentation, practice and reflection.
- Collaborative learning is planned through the use of Talking Partners, group work and whole class discussion. Children investigate the different ways to solve problems and discuss effective methods.

- Concepts and lesson content are carefully sequenced in order that children can make connections, use prior learning and deepen their understanding.
- Sufficient time is spent on blocks of work and lessons are adapted where needed so learning can be sustained rather than having to revisit or reteach concepts.
- Intervention is focused within the lesson through scaffolding, resources and images so pupils can keep up, not catch up. Misconceptions are addressed before the start of the next lesson, if identified through marking.
- Children continue learning number facts for all 4 calculations to increase fluency so they can be used quickly for calculations.

Assessment and Marking

- Marking is completed against the year group guidance document.
- Next steps may not always be necessary as the next lesson is normally the next steps in learning. Year group colleagues discuss progress, understanding, methods and strategies and adapt lessons accordingly. Responsive planning is used to ensure the needs of all children are met.
- Formative assessment is conducted through effective questioning to assess knowledge and understanding and marking both within the lesson and after.

Calculation methods

- This document describes the expected progression in methods and is designed to ensure consistency within our schools. The policy details the statutory objectives for addition, subtraction, multiplication and division from the revised National Curriculum 2013.
- Progression in written methods of calculation is shown by year group according to the national curriculum requirements, however, children are introduced to methods when they become ready and have developed the requisite skills and knowledge.
- Successful calculations must be underpinned by the appropriate knowledge of number facts, along with the necessary mental skills that are needed to carry out the process and to judge if the calculation was successful.
- The statutory requirements for number, place value and fractions for each year group are detailed in appendix 1.
- By the end of Year 6 children should be equipped with, and are able to choose from, the most appropriate of a range of mental, informal written and formal written methods.
- The national curriculum 2013 states that calculators should not be used as a substitute for good written and mental arithmetic, and therefore recommends they should only be introduced near the end of key stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure.
- Although this policy exemplifies written procedures, the ability to calculate mentally lies at the heart of mathematics: it is taught systematically every day and is the essential basis of the subject. We use 'Big maths resources' from year 1 to year 6 to support pupil's knowledge, understanding and quick recall of number facts. In addition, this allows teaching staff to identify and address any misconceptions or areas for revision.

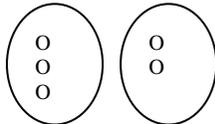
Addition

Foundation

-Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number.
 -Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
 -They solve problems, including doubling, halving and sharing.

Developing spoken language including: Add, more, altogether, count on, total, one more, two more, ten more. Songs, rhymes, dice and number games
 Number stories for combining two sets of objects:
 e.g.If there are 3 pigs in a field, and 2 in a sty how many are there altogether?

Teacher models $3+2=5$ with practical resources.



Numicon is used to add 2 numbers and to understand number bonds for numbers up to 5 and then 10



Children make the two numbers out of straws and combine them. Straws can be bundled into tens to introduce place value of two digit numbers.



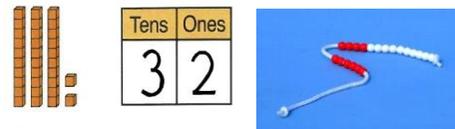
At this stage the Cuisenaire rods can be used through play and to develop the language needed for the bar method – longer than, shorter than, equal to, same as, ? add ? is equal to ?



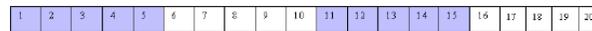
Year 1

-Read, write and interpret mathematical statements involving addition (+) and (=) signs.
 -Represent and use number bonds within 20
 -Add and one-digit and two-digit numbers to 20, including zero
 -Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems such as $4 + \square = 7$

Songs and rhymes
 Working with apparatus, including Numicon and straws continues. Bead strings and base 10 resources are introduced to support addition of one and two digit numbers.

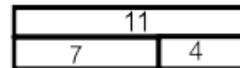
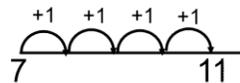


Use + and = signs. Putting the larger number first $13+3=$ and counting on along a number track and/or 100 grid. Counting on in 10s and 1s with the support of resources and 100 grid.



Introduction of empty number lines (ENL) and bar model as a way to represent calculations alongside number tracks and practical resources.

$$7 + 4 = 11$$



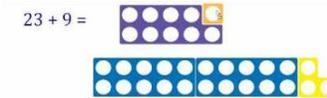
Number bonds of all numbers to 20 and missing number problems are solved with practical resources and counting on along a number line.

$$\square + 7 = 12 \quad 9 + \square = 18$$

Year 2

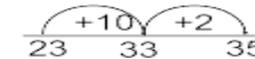
-Solve addition problems using concrete objects and pictorial representations, including:
 • a two-digit number and ones
 • a two-digit number and tens
 • two two-digit numbers
 • adding three one-digit numbers
 -Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
 -Show that addition of two numbers can be done in any order (commutative)
 -Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems.
 -Start to record addition in columns.

Counting in 10s from any number
 Rapid recall of all number bonds for all numbers to 20



Structured numberlines to 100
 Empty Number Lines (ENL) – Use efficient jumps by partitioning numbers into tens and ones.

$$23+12=$$



Using mental strategies such as:
 Add 9 or 11 by adding 10 and adjusting (-1 or +1)
 Near doubles $14 + 15$ is $15 + 15 = 30 - 1$

Start to record addition in columns, using expanded methods

$$40 + 7$$

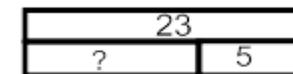
$$30 + 6$$

$$70 + 13 = 83 \quad \text{start by adding ones}$$

Can check by using the inverse operation.

Use to solve missing number problems

$$\text{Eg } \square + 5 = 23 \quad (\text{I know } 23 - 5 = 18)$$



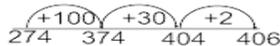
Addition

Year 3

- Add a range of numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- Add numbers with up to three digits, using formal written methods of columnar addition
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Add fractions with the same denominator within one whole (for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)

Use partitioning to support mental calculations up to three digit numbers. Using an empty number line to count on.

$$274 + 132$$



Continue mental strategies as in Year 2 with appropriate numbers e.g. $350 + 189$ is the same as $350 + 190 - 1$.

Extend use of columnar addition, using base 10 materials to model, starting without exchanging, then introduce exchange in one column at a time.

$$\begin{array}{r} 374 \\ 123 + \\ \hline 497 \end{array} \quad \begin{array}{r} 374 \\ 18 + \\ \hline 392 \end{array} \quad \begin{array}{r} 374 \\ 351 + \\ \hline 725 \end{array}$$

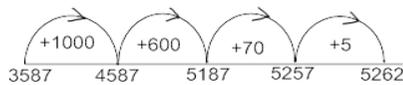
Use manipulatives and images to add fractions with the same denominator ($\frac{3}{5} + \frac{2}{5} = \frac{5}{5}$)



Year 4

- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- Add fractions with the same denominator
- Solve simple measure and money problems involving fractions and decimals to two decimal places

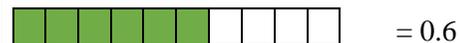
Using an empty number line to count on. $3587 + 1675$



Extend use of columnar addition to 4 digit numbers with exchange in different columns, using base 10 materials and place value counters to support.

$$\begin{array}{r} 1279 \\ 2243 + \\ \hline 3522 \\ 11 \end{array}$$

Extend to adding fractions and decimals, using Numicon, multilink, cuisinaire and decimal strips to support.



Year 5

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Add numbers mentally with increasingly large numbers (eg. $8\,462 + 2300 = 10\,762$).
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why.
- Add and subtract fractions with the same denominator and denominators that are multiples of the same number

Base 10 materials and place value counters continue to support formal columnar addition for numbers with more than 4 digits, including exchanging.

$$\begin{array}{r} 21271 \\ 12243 + \\ \hline 33514 \\ 1 \end{array}$$



Extend to decimals.

$$\begin{array}{r} 42.432 \\ 12.713 + \\ \hline 55.145 \\ 1 \end{array}$$



Add fractions with the same denominator and multiples of the same number.

$$\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

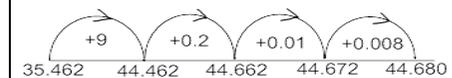


Use of arrays to represent addition of fractions

Year 6

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Perform mental calculations, including with mixed operations and large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Using an empty number line to count on. $35.462 + 9.218$



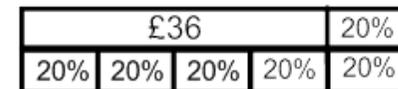
Use formal column addition for large numbers (>1 million)

$$\begin{array}{r} 2353248 \\ 1254173 + \\ \hline 3607421 \\ 1 \quad 11 \end{array}$$

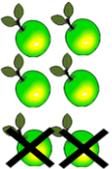
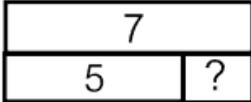
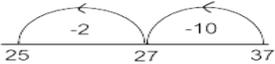
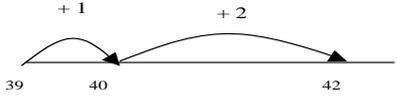
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

$$\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$$

Problem solving - There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?



$$36 \div 4 = 9 \text{ (20\%)} \text{ so } £36 + £9 = £45$$

Subtraction										
Foundation	Year 1	Year 2								
<p>-Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number.</p> <p>-Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.</p> <p>-They solve problems, including doubling, halving and sharing.</p>	<p>-Read, write and interpret mathematical statements involving addition (-) and (=) signs.</p> <p>-Represent and use number bonds and related subtraction facts within 20</p> <p>-Subtract one-digit and two-digit numbers to 20, including zero</p> <p>-Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems such as $8 - \square = 5$</p>	<p>-Solve subtraction problems using concrete objects and pictorial representations, including:</p> <ul style="list-style-type: none"> a two-digit number and ones a two-digit number and tens two two-digit numbers <p>-Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100</p> <p>-Show that subtraction of one number from another cannot be done in any order.</p> <p>-Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems.</p> <p>-Start to record subtraction in columns.</p>								
<p>Developing spoken language including: Take (away), leave, how many are left/left over? How many have gone? One less, two less, ten less, how many fewer is..?</p> <p>Songs and rhymes Dice and number games. Counting back with a number line as a visual support.</p>  <p>Number stories using objects, counters, multilink and straws.</p>  <p>How many are there? How many now? (after some have been removed) Teacher verbally modelling number sentences as 6 take away 2 is 4 and how to record it as $6 - 2 = 4$</p>	<p>Working with apparatus, including multilink and straws continues. Bead strings and base 10 resources are introduced to support subtraction of one and two digit numbers.</p>  <p>Songs and rhymes to support language – take away, fewer, less</p> <p>Physical and practical work on number lines and 100 grids, jumping backwards in 10s and 1s.</p> <p>Number stories, 15 people on a bus 3 get off, how many are left on?</p> <p>$9 - \square = 4$ $\square - 5 = 4$</p> <p>Finding the difference by comparing quantities, supported by the use of a number line or bar model to visualise the difference.</p>  	<p>Counting back in 10s from any number to 100 Jumping back on empty number line (ENL) by partitioning into tens and ones</p> <p>37-12</p>  <p>Continue finding the difference between numbers with practical resources, leading to using the ENL for numbers that are close together.</p> <p>42-39=</p>  <p>Use addition as the inverse operation to check and solve missing number problems eg $\square - 8 = 4$ (I know $8 + 4 = 12$)</p> <p>Janie has 40 beads. She loses 25 of them. How many does she have left?</p> <table border="1" data-bbox="1420 1129 1756 1219"> <tr> <td>10</td> <td>10</td> <td>10</td> <td>10</td> </tr> <tr> <td>10</td> <td>10</td> <td>5</td> <td>?</td> </tr> </table> <p>Start to record subtraction in columns, using expanded methods alongside practical resources.</p> $\begin{array}{r} 40 \ 5 \\ \underline{20 \ 3} \\ 20 \ 2 \end{array}$ <p>$45 - 23 = 22$</p>	10	10	10	10	10	10	5	?
10	10	10	10							
10	10	5	?							

Subtraction

Year 3

-Subtract a range of numbers mentally, including:

- a three-digit number and ones
- three-digit number and tens
- a three-digit number and hundreds

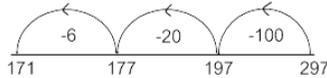
-Subtract numbers with up to three digits, using formal written methods of columnar subtraction

-Estimate the answer to a calculation and use inverse operations to check answers

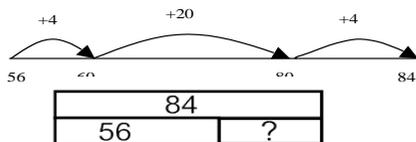
-Solve problems, including missing number problems, using number facts, place value, and more complex addition.

-Subtract fractions with the same denominator within one whole (for example, $\frac{5}{7} - \frac{1}{7} = \frac{4}{7}$)

Develop confidence in counting back in 100s, 10s and 1s from any number.
Use an empty number line to count back.
 $297 - 126 =$



Count on to find the difference using empty numberline $84 - 56 =$



Use base 10 materials to explain the expanded method of decomposition for numbers too large to do mentally. Start with calculations without exchanging, then introduce exchange with one column.
 $572 - 158 =$

$$\begin{array}{r} 500 \\ 100 \\ 400 \\ \hline 60 \\ 50 \\ 10 \\ \hline 12 \\ 8 \\ 4 \\ \hline 61 \\ 158 \\ 414 \end{array}$$

Year 4

-Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

-Estimate and use inverse operations to check answers to a calculation

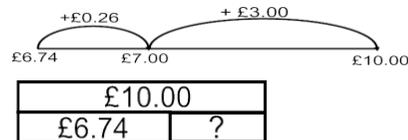
-Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

-Subtract fractions with the same denominator

-Solve simple measure and money problems involving fractions and decimals to two decimal places

Use an empty number line to both count back, and find the difference between two numbers by counting on (particularly useful to find change). (Up to 4 digits)

How much change will I get from £10 if my shopping costs £6.74?



Begin to use decimals to 2 decimal places.

Use expanded method of decomposition, with 4 digit numbers and exchanging, leading to more compact recording.
Use base 10 materials and place value counters to support understanding
 $2367 - 749 =$

$$\begin{array}{r} 1000 \\ 2000 \\ 1000 \\ \hline 1300 \\ 300 \\ 600 \\ \hline 50 \\ 60 \\ 10 \\ \hline 17 \\ 7 \\ 9 \\ \hline 1151 \\ 749 \\ 1618 \end{array}$$

Begin to extend to decimals.

Year 5

-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)

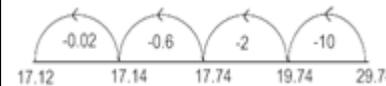
-Subtract numbers mentally with increasingly large numbers (eg. $10\,462 - 2300 = 8\,162$).

-Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

-Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why.

-Add and subtract fractions with the same denominator and denominators that are multiples of the same number

Using an empty number line to count on and back, choosing the most efficient method, including finding the difference between two numbers (whole and decimal numbers)
 $29.74 - 12.62 =$

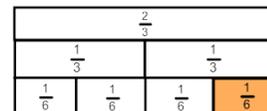


Use formal written columnar subtraction including for decimals. Continue using base 10 and place value counters to represent exchanging.

$$\begin{array}{r} 21 \\ 26.57 \\ 17.46 \\ \hline 19.11 \end{array}$$

Subtract fractions with the same denominator and multiples of the same number.

$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6}$$



Year 6

-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)

-Perform mental calculations, including with mixed operations and large numbers

-Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

-Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

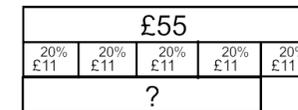
Use a formal written method of columnar subtraction.

$$\begin{array}{r} 21 \\ 26.573 \\ 18.462 \\ \hline 18.111 \end{array}$$

Apply to problem solving contexts e.g. money and measures

There is 20% off in a sale. How much would a track suit cost if the normal price was £55?

$$£55 \div 5 = £11 \quad £55 - £11 = £44$$



Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

$$\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$$

Multiplication

Foundation

Year 1

Year 2

-Start to solve problems involving doubling.

-Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
-Make connections between arrays, number patterns, and counting in twos, fives and tens.

-Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
-Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (\times) and equals (=) signs
-Show that multiplication of two numbers can be done in any order (commutative)
-Solve problems involving multiplication using materials, arrays, repeated addition, mental methods and multiplication and including problems in contexts.

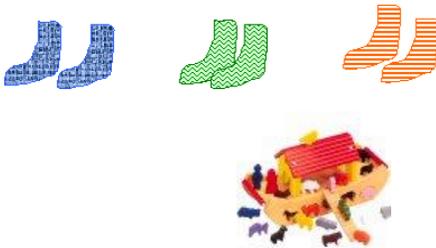
Counting in ones, twos, tens

Counting in twos, fives and tens
Knowing doubles of numbers to 10
Dice and domino games with doubles

Counting in 3s

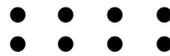
Doubles of all numbers up to 10 and doubles of multiples of 10 to 100
Recognise odd and even numbers, supported by Numicon.

Odd and even numbers
Matching pairs eg socks
Noahs ark



Finding patterns of numbers using a 100 square and make connections with arrays.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Repeated addition of sets of objects, teacher modelling $2+2+2 = 6$
Use coins for repeated addition



and model using Numicon.



$$5 + 5 + 5 = 15$$

Begin to introduce the language; multiply, times, sets or groups of, number of rows and total.

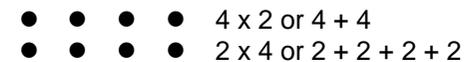
Odd.



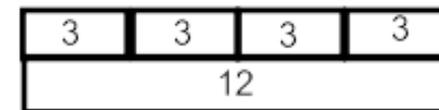
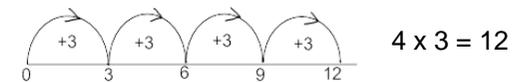
Even.



Arrays and repeated addition



Counting on in groups on an empty numberline



Multiplication

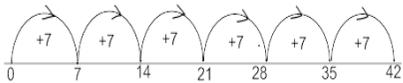
Year 3

-Recall and use multiplication and division facts for the 3, 4 and 9 multiplication tables
 -Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods of short multiplication
 -Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

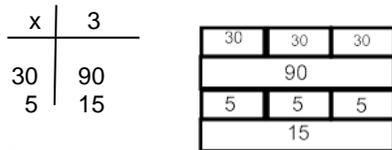
Multiply 1 and 2 digit numbers by 10
 $34 \times 10 = 340$
 Multiply multiples of 10 using known x tables
 $60 \times 3 = 180$

Know that division is inverse of multiplication

Understand multiplication as repeated addition
 Use a number line to solve 6×7



Continue to use arrays and the bar method and to link to using the grid method to solve 35×3



$90 + 15 = 105$

Progress towards formal short multiplication by comparing grid and formal method

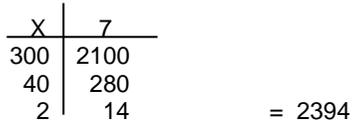
$$\begin{array}{r} 35 \\ \times 3 \\ \hline 15 \\ 90 \\ \hline 105 \end{array}$$

Year 4

-Recall multiplication and division facts for multiplication tables up to 12×12
 -Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 and multiplying together three numbers
 -Recognise and use factor pairs and commutativity in mental calculations
 -Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
 -Solve problems involving multiplying including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Multiply by using known facts. e.g. To multiply by 20, multiply by 10 then double
 Multiply up to 3 digit numbers by 10 or 100
 Multiply multiples of 10 and 100 using x tables

Use the grid method $342 \times 7 =$

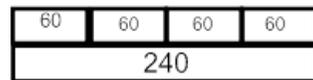


Extend use of formal short multiplication
 342×7 becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ 21 \\ \hline \end{array}$$

Use methods within problem solving contexts such as money and measures. Eg apply scaling to problems such as recipes and ingredients.

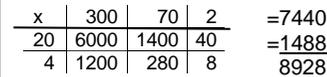
4 children drink 59ml of juice each. How much do they drink altogether? $4 \times 59 =$
 59 is 1 less than 60 so $4 \times 60 = 240 - 4 = 236$



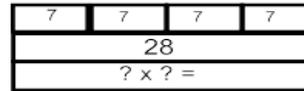
Year 5

-Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
 -Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
 -Multiply numbers mentally drawing upon known facts, including multiplying whole numbers and those involving decimals by 10, 100 and 1000
 -Recognise and use square numbers and cube numbers, write the notation for both $(^2)$ and $(^3)$ and solve problems involving multiplication using knowledge of factors and multiples, squares and cubes
 -Solve problems involving scaling by simple fractions.
 -Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Grid method HTU X TU, approximate first
 $(400 \times 20 = 8000)$

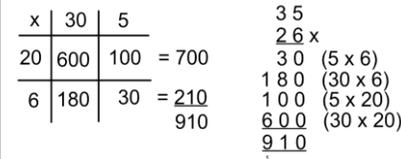


Missing number type problems $_x_ = 4 \times 7$, by using knowledge of factors that make 28



Multiply decimals and integers by 10, 100 and 1000.

Use formal extended long multiplication for up to 4 digit x 2 digit. Make connections back to grid method. Develop restricted long multiplication method where suitable.



Multiply proper fractions e.g. $\frac{1}{2} \times \frac{2}{5}$



Year 6

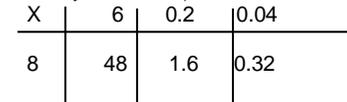
-Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
 -Multiply one-digit numbers with up to two decimal places by whole numbers
 -Perform mental calculations, including with mixed operations and large numbers
 -Multiply simple pairs of proper fractions
 -Identify common factors, common multiples and prime numbers

Use formal long multiplication for up to 4 digits x 2 digits. Eg 1354×24

$$\begin{array}{r} 1354 \\ \times 24 \\ \hline 5416 \\ 27080 \\ \hline 32496 \end{array}$$

Extend to decimals.

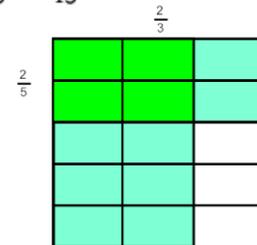
Use the grid method to solve 6.24×8 (approximately $6 \times 8 = 48$)



$= 49.92$

Multiply pairs of proper fractions.

$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

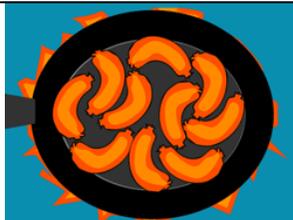


Division

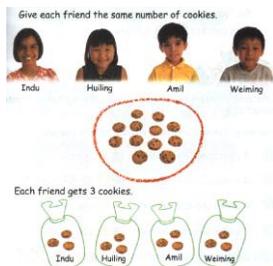
Foundation

-Start to solve problems involving halving and sharing

Practical activities, songs and rhymes.
10 fat sausages.



Sharing during snack time by giving 1 each
Is there an easier way of sharing a larger amount?
Eg 2 at a time



Making groups/piles of 2, finding partners
Eg in PE grouping in 2s, how many pairs are there?
1 ball for each pair, how many balls do I need to get out?

Year 1

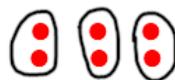
-Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Practical activities, songs, and games.

Sharing – 6 sweets are shared between 2 people.
How many do they have each?



Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)



Cutting cakes, pizza in half, sharing related to fractions

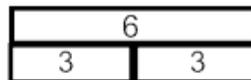
Finding half of a group of objects



Knowing halves of even numbers to 20



Use Numicon and the bar method to represent



Year 2

-Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
-Calculate mathematical statements for division within the multiplication tables and write them using the division (\div) and equals (=) signs
-Show that division of one number by another cannot be done in any order
-Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.

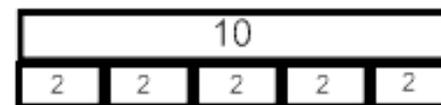
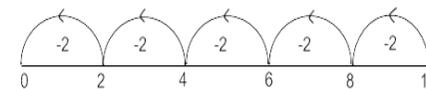
Relate division to fractions $\frac{1}{2}$ or $\frac{1}{4}$ of 12, 20. Half of 12 is $12 \div 2 =$

Understand division as sharing and grouping
18 shared between 3



Counting on and back in 2s, 5s, 10s
How many 2s in 10?

Teach grouping along a numberline or using the bar method

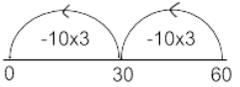
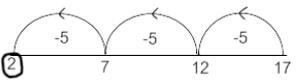
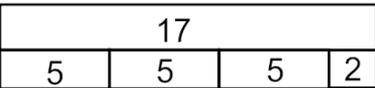
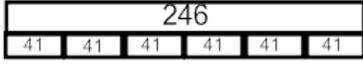
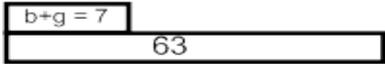


How many 2s in 10?

\div = signs and missing numbers

$6 \div 2 = \square$	$\square = 6 \div 2$
$6 \div \square = 3$	$3 = 6 \div \square$
$\square \div 2 = 3$	$3 = \square \div 2$
$\square \div \nabla = 3$	$3 = \square \div \nabla$

Division

Year 3	Year 4	Year 5	Year 6
<p>-Recall and use multiplication and division facts for the 3, 4 and 9 multiplication tables</p> <p>-Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to formal written methods</p> <p>-Solve problems, including missing number problems, involving multiplication and division</p> <p>-Calculate simple remainders after division</p>	<p>-Recall multiplication and division facts for multiplication tables up to 12×12</p> <p>-Recognise and use factor pairs in mental calculations</p> <p>-Divide two-digit and three-digit numbers by a one-digit number using formal written layout</p> <p>-Divide a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths</p>	<p>-Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p> <p>-Divide whole numbers and those involving decimals by 10, 100 and 1000</p> <p>-Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers</p> <p>-Establish whether a number up to 100 is prime and recall prime numbers up to 19</p>	<p>-Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p> <p>-Identify common factors, common multiples and prime numbers</p> <p>-Divide proper fractions by whole numbers (for example, $\frac{1}{3} \div 2 = \frac{1}{6}$)</p> <p>-Associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $\frac{3}{8}$)</p>
<p>$\frac{1}{4}$ or $\frac{1}{2}$ of 24, 60, 200.</p> <p>Understand division as grouping and as sharing.</p> <p>Use repeated subtraction to find how many groups. E.g. for $60 \div 3$ How many groups of 3 are there in 60?</p> <p>Start to use more efficient 'chunking' methods on an ENL</p>  <p>Introduce remainders $17 \div 5 = 3r2$</p>   <p>Make clear links between \times and \div and find missing numbers using known multiplication facts.</p> <p>$45 \div \square = 5$</p> <p>Divide by 10 introducing one decimal place $650 \div 10 = 65$ $72 \div 10 = 7.2$</p>	<p>Continue to understand division as both sharing and grouping.</p> <p>A jug holds 250ml of water. How many jugs will I be able to fill with 10L of water?</p>  <p>1L can fill 4 jugs so 10L will fill 40 jugs.</p> <p>Approximate first. Use informal methods relating to the child's mental methods and within \times tables known, including those with remainders.</p> <p>$50 \div 6 = 8r2$</p> <p>Extend to larger numbers and written methods using 'chunking'.</p> <p>$72 \div 5$</p> <p>- $\frac{50}{22}$ 10 $\times 5$</p> <p>- $\frac{20}{2}$ 4 $\times 5$</p> <p>Answer : 14 remainder 2</p> <p>Divide a 1 or 2 digit number by 10 and 100</p> <p>$4 \div 100 = 0.04$ $27 \div 100 = 0.27$</p>	<p>Develop written methods such as:</p> <p>- 'chunking' making the chunks more efficient by using known multiplication facts. $256 \div 7 =$</p> <p>$\frac{256}{46}$ 30 $\times 7$</p> <p>- $\frac{42}{4}$ 6 $\times 7$ = 36 r4</p> <p>Use representations such as the bar method to check or prove a calculation.</p> <p>There are 246 children on a school trip but they must be split into 6 equal groups. $246 \div 6 =$</p> <p>$6 \times 40 = 240$ $6 \times 1 = 6$ $40+1=41$</p>  <p>- Introduce more formal long and short division methods making links to chunking as above</p> <p>$\begin{array}{r} 36\text{ r}4 \\ 7 \overline{) 256} \\ \underline{21} \\ 46 \\ \underline{42} \\ 4 \end{array}$ $\begin{array}{r} 36\text{ r}4 \\ 7 \overline{) 256} \end{array}$</p>	<p>Continue to use 'chunking' method dividing by 2 digit numbers. $977 \div 36 =$</p> <p>$\frac{977}{617}$ 10 $\times 36$ - $\frac{720}{257}$ 20 $\times 36$</p> <p>- $\frac{360}{257}$ 10 $\times 36$ refine to $\frac{180}{77}$ 5 $\times 36$</p> <p>- $\frac{180}{77}$ 5 $\times 36$ - $\frac{72}{5}$ 2 $\times 36$</p> <p>- $\frac{72}{5}$ 2 $\times 36$ Answer: $27 \frac{5}{36}$</p> <p>Quotients may be expressed as fractions or decimal fractions up to 2 decimal places. Use formal long and short division methods with larger numbers.</p> <p>$\begin{array}{r} 212.29 \\ 24 \overline{) 5095.00} \\ \underline{48} \\ 29 \\ \underline{24} \\ 55 \\ \underline{48} \\ 70 \\ \underline{48} \\ 220 \\ \underline{216} \end{array}$ $\begin{array}{r} 212.29 \\ 24 \overline{) 5095.00} \end{array}$</p> <p>$\begin{array}{r} 212\text{ r}7 \\ 24 \overline{) 5095} \end{array}$</p> <p>At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there? $63 \div 7 = 9$ $4 \times 9 = 36$</p> 

Appendix 1. Number, place value and fractions statutory requirements (National Curriculum 2013, Early Years Foundation Stage 2012).

Foundation stage.

Pupils should be taught to:

- children count reliably with numbers from 1 to 20
- place the numbers in order and say which number is one more or one less than a given number.

Year 1.

- Pupils should be taught to:
- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in numerals and words
- recognise, find and name a half as one of two equal parts of an object, shape or quantity
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Year 2.

Pupils should be taught to:

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use $<$, $>$ and $=$ signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems
- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.

Year 3.

Pupils should be taught to:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations

- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas
- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

Year 4.

Pupils should be taught to

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value
- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

Year 5.

Pupils should be taught to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000

- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals
- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5}$]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

Year 6.

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above
- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form (for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$)
- divide proper fractions by whole numbers (for example, $\frac{1}{3} \div 2 = \frac{1}{6}$)
- associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $\frac{3}{8}$)
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.